

Excited heavy tetraquarks with hidden charm

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The masses of the excited heavy tetraquarks with hidden charm are calculated within the relativistic diquark-antidiquark picture. The dynamics of the light quark in a heavy-light diquark is treated completely relativistically. The diquark structure is taken into account by calculating the diquark-gluon form factor. New experimental data on charmonium-like states above open charm threshold are discussed. The obtained results indicate that $X(3872)$, $Y(4260)$, $Y(4360)$, $Z(4248)$, $Z(4433)$ and $Y(4660)$ could be tetraquark states with hidden charm.

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Recently, significant experimental progress has been achieved in charmonium spectroscopy. Several new states, such as $X(3872)$, $Y(4260)$, $Y(4360)$, $Y(4660)$, $Z(4248)$, $Z(4430)$, etc., were observed [1] which cannot be simply accommodated in the quark-antiquark ($c\bar{c}$) picture. These states and especially the charged ones can be considered as indications of the possible existence of exotic multiquark states [2, 3]. In our papers [4, 5] we calculated masses of the ground state heavy tetraquarks in the framework of the relativistic quark model based on the quasipotential approach in quantum chromodynamics. Here we extend this analysis to the consideration of the excited tetraquark states with hidden charm. As previously, we use the diquark-antidiquark picture to reduce a complicated relativistic four-body problem to the subsequent two more simple two-body problems. The first step consists in the calculation of the masses, wave functions and form factors of the diquarks, composed from light and heavy quarks. At the second step, a heavy tetraquark is considered to be a bound diquark-antidiquark system. It is important to emphasize that we do not consider the diquark as a point particle but explicitly take into account its structure by calculating the form factor of the diquark-gluon interaction in terms of the diquark wave functions.

In the quasipotential approach and diquark-antidiquark picture of heavy tetraquarks the interaction of two quarks in a diquark and the diquark-antidiquark interaction in a tetraquark are described by the diquark wave function (Ψ_d) of the bound quark-quark state and by the tetraquark wave function (Ψ_T) of the bound diquark-antidiquark state, respectively. These wave functions satisfy the quasipotential equation of the Schrödinger type [6]

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R} \right) \Psi_{d,T}(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}; M) \Psi_{d,T}(\mathbf{q}), \quad (1)$$

where the relativistic reduced mass is

$$\mu_R = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}, \quad (2)$$

and E_1, E_2 are given by

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}. \quad (3)$$

Here, $M = E_1 + E_2$ is the bound-state mass (diquark or tetraquark), $m_{1,2}$ are the masses of quarks (q and Q) which form the diquark or of the diquark (d) and antiquark (\bar{d}) which form the heavy tetraquark (T), and \mathbf{p} is their relative momentum. In the center-of-mass system the relative momentum squared on mass shell reads

$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}. \quad (4)$$

The kernel $V(\mathbf{p}, \mathbf{q}; M)$ in Eq. (1) is the quasipotential operator of the quark-quark or diquark-antidiquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive-energy states. In the following analysis we closely follow the similar construction of the quark-antiquark interaction in mesons which were extensively studied in our relativistic quark model [6, 7]. For the quark-quark interaction in a diquark we use the relation $V_{qq} = V_{q\bar{q}}/2$ arising under the assumption of an octet structure of the interaction from the difference in the qq and $q\bar{q}$ colour states. An important role in this construction is played by the Lorentz structure of the confining interaction. In our analysis of mesons, while constructing the quasipotential of the quark-antiquark interaction, we assumed that the effective interaction is the sum of the usual one-gluon exchange term and a mixture of long-range vector and scalar linear confining potentials, where the vector confining potential contains the Pauli term. We use the same conventions for the construction of the quark-quark and diquark-antidiquark interactions in the tetraquark. The quasipotential is then defined as follows [7, 8].

(a) For the quark-quark (Qq) interactions, $V(\mathbf{p}, \mathbf{q}; M)$ reads

$$V(\mathbf{p}, \mathbf{q}; M) = \bar{u}_1(p)\bar{u}_2(-p)\mathcal{V}(\mathbf{p}, \mathbf{q}; M)u_1(q)u_2(-q), \quad (5)$$

with

$$\mathcal{V}(\mathbf{p}, \mathbf{q}; M) = \frac{1}{2} \left[\frac{4}{3} \alpha_s D_{\mu\nu}(\mathbf{k}) \gamma_1^\mu \gamma_2^\nu + V_{\text{conf}}^V(\mathbf{k}) \Gamma_1^\mu(\mathbf{k}) \Gamma_{2;\mu}(-\mathbf{k}) + V_{\text{conf}}^S(\mathbf{k}) \right].$$

Here, α_s is the QCD coupling constant; $D_{\mu\nu}$ is the gluon propagator in the Coulomb gauge,

$$D^{00}(\mathbf{k}) = -\frac{4\pi}{\mathbf{k}^2}, \quad D^{ij}(\mathbf{k}) = -\frac{4\pi}{k^2} \left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2} \right), \quad D^{0i} = D^{i0} = 0, \quad (6)$$

and $\mathbf{k} = \mathbf{p} - \mathbf{q}$; γ_μ and $u(p)$ are the Dirac matrices and spinors,

$$u^\lambda(p) = \sqrt{\frac{\epsilon(p) + m}{2\epsilon(p)}} \left(\frac{1}{\frac{\boldsymbol{\sigma} \cdot \mathbf{p}}{\epsilon(p) + m}} \right) \chi^\lambda, \quad (7)$$

with $\epsilon(p) = \sqrt{\mathbf{p}^2 + m^2}$.

The effective long-range vector vertex of the quark is defined [7] by

$$\Gamma_\mu(\mathbf{k}) = \gamma_\mu + \frac{i\kappa}{2m}\sigma_{\mu\nu}\tilde{k}^\nu, \quad \tilde{k} = (0, \mathbf{k}), \quad (8)$$

where κ is the Pauli interaction constant characterizing the anomalous chromomagnetic moment of quarks. In configuration space the vector and scalar confining potentials in the nonrelativistic limit [9] reduce to

$$\begin{aligned} V_{\text{conf}}^V(r) &= (1 - \varepsilon)V_{\text{conf}}(r), \\ V_{\text{conf}}^S(r) &= \varepsilon V_{\text{conf}}(r), \end{aligned} \quad (9)$$

with

$$V_{\text{conf}}(r) = V_{\text{conf}}^S(r) + V_{\text{conf}}^V(r) = Ar + B, \quad (10)$$

where ε is the mixing coefficient.

(b) For the diquark-antidiquark ($d\bar{d}'$) interaction, $V(\mathbf{p}, \mathbf{q}; M)$ is given by

$$\begin{aligned} V(\mathbf{p}, \mathbf{q}; M) &= \frac{\langle d(P)|J_\mu|d(Q)\rangle}{2\sqrt{E_d E_d}} \frac{4}{3} \alpha_s D^{\mu\nu}(\mathbf{k}) \frac{\langle d'(P')|J_\nu|d'(Q')\rangle}{2\sqrt{E_{d'} E_{d'}}} \\ &\quad + \psi_d^*(P)\psi_{d'}^*(P') \left[J_{d;\mu} J_{d'}^\mu V_{\text{conf}}^V(\mathbf{k}) + V_{\text{conf}}^S(\mathbf{k}) \right] \psi_d(Q)\psi_{d'}(Q'), \end{aligned} \quad (11)$$

where $\langle d(P)|J_\mu|d(Q)\rangle$ is the vertex of the diquark-gluon interaction which takes into account the finite size of the diquark and is discussed below $[P^{(\prime)} = (E_{d^{(\prime)}}, \pm\mathbf{p})$ and $Q^{(\prime)} = (E_{d^{(\prime)}}, \pm\mathbf{q})$, $E_d = (M^2 - M_{d'}^2 + M_d^2)/(2M)$ and $E_{d'} = (M^2 - M_d^2 + M_{d'}^2)/(2M)$].

The diquark state in the confining part of the diquark-antidiquark quasipotential (11) is described by the wave functions

$$\psi_d(p) = \begin{cases} 1 & \text{for a scalar diquark,} \\ \varepsilon_d(p) & \text{for an axial-vector diquark,} \end{cases} \quad (12)$$

where the four-vector

$$\varepsilon_d(p) = \left(\frac{(\boldsymbol{\varepsilon}_d \cdot \mathbf{p})}{M_d}, \boldsymbol{\varepsilon}_d + \frac{(\boldsymbol{\varepsilon}_d \cdot \mathbf{p})\mathbf{p}}{M_d(E_d(p) + M_d)} \right), \quad \varepsilon_d^\mu(p)p_\mu = 0, \quad (13)$$

is the polarization vector of the axial-vector diquark with momentum \mathbf{p} , $E_d(p) = \sqrt{\mathbf{p}^2 + M_d^2}$, and $\varepsilon_d(0) = (0, \boldsymbol{\varepsilon}_d)$ is the polarization vector in the diquark rest frame. The effective long-range vector vertex of the diquark can be presented in the form

$$J_{d;\mu} = \begin{cases} \frac{(P+Q)_\mu}{2\sqrt{E_d E_d}} & \text{for a scalar diquark,} \\ -\frac{(P+Q)_\mu}{2\sqrt{E_d E_d}} + \frac{i\mu_d}{2M_d} \Sigma_\mu^\nu \tilde{k}_\nu & \text{for an axial-vector diquark.} \end{cases} \quad (14)$$

Here, the antisymmetric tensor Σ_μ^ν is defined by

$$(\Sigma_{\rho\sigma})_\mu^\nu = -i(g_{\mu\rho}\delta_\sigma^\nu - g_{\mu\sigma}\delta_\rho^\nu), \quad (15)$$

TABLE I: Masses M and form factor parameters of charmed diquarks. S and A denote scalar and axial vector diquarks which are antisymmetric $[\cdots]$ and symmetric $\{\cdots\}$ in flavour, respectively.

Quark content	Diquark type	M (MeV)	ξ (GeV)	ζ (GeV ²)
$[c, q]$	S	1973	2.55	0.63
$\{c, q\}$	A	2036	2.51	0.45
$[c, s]$	S	2091	2.15	1.05
$\{c, s\}$	A	2158	2.12	0.99

and the axial-vector diquark spin \mathbf{S}_d is given by $(S_{d;k})_{il} = -i\varepsilon_{kil}$; μ_d is the total chromomagnetic moment of the axial-vector diquark.

The constituent quark masses $m_c = 1.55$ GeV, $m_u = m_d = 0.33$ GeV, $m_s = 0.5$ GeV and the parameters of the linear potential $A = 0.18$ GeV² and $B = -0.3$ GeV have values typical in quark models. The value of the mixing coefficient of vector and scalar confining potentials $\varepsilon = -1$ has been determined from the consideration of charmonium radiative decays [6] and the heavy-quark expansion [10]. The universal Pauli interaction constant $\kappa = -1$ has been fixed from the analysis of the fine splitting of heavy quarkonia 3P_J - states [6]. In this case, the long-range chromomagnetic interaction of quarks vanishes in accordance with the flux-tube model.

At the first step, we calculate the masses and form factors of the heavy-light diquark. As it is well known, the light quarks are highly relativistic, which makes the v/c expansion inapplicable and thus, a completely relativistic treatment of the light quark dynamics is required. To achieve this goal, we closely follow our consideration of diquarks in heavy baryons and adopt the same procedure to make the relativistic potential local by replacing $\epsilon_{1,2}(p) = \sqrt{m_{1,2}^2 + \mathbf{p}^2} \rightarrow E_{1,2} = (M^2 - m_{2,1}^2 + m_{1,2}^2)/2M$. Solving numerically the quasipotential equation (1) with the complete relativistic potential, which depends on the diquark mass in a complicated highly nonlinear way [11], we get the diquark masses and wave functions. In order to determine the diquark interaction with the gluon field, which takes into account the diquark structure, we calculate the corresponding matrix element of the quark current between diquark states. Such calculation leads to the emergence of the form factor $F(r)$ entering the vertex of the diquark-gluon interaction [11]. This form factor is expressed through the overlap integral of the diquark wave functions. Our estimates show that this form factor can be approximated with a high accuracy by the expression

$$F(r) = 1 - e^{-\xi r - \zeta r^2}. \quad (16)$$

The values of the masses and parameters ξ and ζ for heavy-light scalar diquark $[\cdots]$ and axial vector diquark $\{\cdots\}$ ground states are given in Table I.

At the second step, we calculate the masses of heavy tetraquarks considered as the bound states of a heavy-light diquark and antidiquark. For the potential of the diquark-antidiquark interaction (11) we get [5]

$$V(r) = \hat{V}_{\text{Coul}}(r) + V_{\text{conf}}(r) + \frac{1}{2} \left\{ \left[\frac{1}{E_1(E_1 + M_1)} + \frac{1}{E_2(E_2 + M_2)} \right] \frac{\hat{V}'_{\text{Coul}}(r)}{r} - \left[\frac{1}{M_1(E_1 + M_1)} \right. \right.$$

$$\begin{aligned}
& + \frac{1}{M_2(E_2 + M_2)} \left[\frac{V'_{\text{conf}}(r)}{r} + \frac{\mu_d}{2} \left(\frac{1}{M_1^2} + \frac{1}{M_2^2} \right) \frac{V'^V_{\text{conf}}(r)}{r} \right] \mathbf{L} \cdot (\mathbf{S}_1 + \mathbf{S}_2) \\
& + \frac{1}{2} \left\{ \left[\frac{1}{E_1(E_1 + M_1)} - \frac{1}{E_2(E_2 + M_2)} \right] \frac{\hat{V}'_{\text{Coul}}(r)}{r} - \left[\frac{1}{M_1(E_1 + M_1)} - \frac{1}{M_2(E_2 + M_2)} \right] \right. \\
& \times \frac{V'_{\text{conf}}(r)}{r} + \frac{\mu_d}{2} \left(\frac{1}{M_1^2} - \frac{1}{M_2^2} \right) \frac{V'^V_{\text{conf}}(r)}{r} \left. \right\} \mathbf{L} \cdot (\mathbf{S}_1 - \mathbf{S}_2) \\
& + \frac{1}{E_1 E_2} \left\{ \mathbf{p} [\hat{V}_{\text{Coul}}(r) + V^V_{\text{conf}}(r)] \mathbf{p} - \frac{1}{4} \Delta V^V_{\text{conf}}(r) + \hat{V}'_{\text{Coul}}(r) \frac{\mathbf{L}^2}{2r} \right. \\
& + \frac{1}{r} \left[\hat{V}'_{\text{Coul}}(r) + \frac{\mu_d}{4} \left(\frac{E_1}{M_1} + \frac{E_2}{M_2} \right) V'^V_{\text{conf}}(r) \right] \mathbf{L} \cdot (\mathbf{S}_1 + \mathbf{S}_2) \\
& + \frac{\mu_d}{4} \left(\frac{E_1}{M_1} - \frac{E_2}{M_2} \right) \frac{V'^V_{\text{conf}}(r)}{r} \mathbf{L} \cdot (\mathbf{S}_1 - \mathbf{S}_2) \\
& + \frac{1}{3} \left[\frac{1}{r} \hat{V}'_{\text{Coul}}(r) - \hat{V}''_{\text{Coul}}(r) + \frac{\mu_d^2}{4} \frac{E_1 E_2}{M_1 M_2} \left(\frac{1}{r} V'^V_{\text{conf}}(r) - V''^V_{\text{conf}}(r) \right) \right] \\
& \times \left[\frac{3}{r^2} (\mathbf{S}_1 \cdot \mathbf{r})(\mathbf{S}_2 \cdot \mathbf{r}) - \mathbf{S}_1 \cdot \mathbf{S}_2 \right] \\
& + \frac{2}{3} \left[\Delta \hat{V}_{\text{Coul}}(r) + \frac{\mu_d^2}{4} \frac{E_1 E_2}{M_1 M_2} \Delta V^V_{\text{conf}}(r) \right] \mathbf{S}_1 \cdot \mathbf{S}_2 \Big\}, \tag{17}
\end{aligned}$$

where

$$\hat{V}_{\text{Coul}}(r) = -\frac{4}{3} \alpha_s \frac{F_1(r) F_2(r)}{r}$$

is the Coulomb-like one-gluon exchange potential which takes into account the finite sizes of the diquark and antidiquark through corresponding form factors $F_{1,2}(r)$. Here, $\mathbf{S}_{1,2}$ and \mathbf{L} are the spin operators of diquark and antidiquark and the operator of the relative orbital angular momentum. In the following we choose the total chromomagnetic moment of the axial-vector diquark $\mu_d = 0$. Such a choice appears to be natural, since the long-range chromomagnetic interaction of diquarks proportional to μ_d then also vanishes in accordance with the flux-tube model.

In the diquark-antidiquark picture of heavy tetraquarks both scalar S (antisymmetric in flavour $(Qq)_{S=0} = [Qq]$) and axial vector A (symmetric in flavour $(Qq)_{S=1} = \{Qq\}$) diquarks are considered. Therefore we get the following structure of the $(Qq)(\bar{Q}\bar{q}')$ ground $(1S)$ states (C is defined only for $q = q'$):

- Two states with $J^{PC} = 0^{++}$:

$$\begin{aligned}
X(0^{++}) &= (Qq)_{S=0}(\bar{Q}\bar{q}')_{S=0} \\
X(0^{++'}) &= (Qq)_{S=1}(\bar{Q}\bar{q}')_{S=1}
\end{aligned}$$

- Three states with $J = 1$:

$$X(1^{++}) = \frac{1}{\sqrt{2}} [(Qq)_{S=1}(\bar{Q}\bar{q}')_{S=0} + (Qq)_{S=0}(\bar{Q}\bar{q}')_{S=1}]$$

TABLE II: Masses of charm diquark-antidiquark ground ($1S$) states (in MeV) calculated in [4]. S and A denote scalar and axial vector diquarks.

State J^{PC}	Diquark content	Mass		
		$cq\bar{c}\bar{q}$	$cs\bar{c}\bar{s}$	$cq\bar{c}\bar{s}$
0^{++}	$S\bar{S}$	3812	4051	3922
$1^{\pm\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	3871	4113	3982
0^{++}	$A\bar{A}$	3852	4110	3967
1^{+-}	$A\bar{A}$	3890	4143	4004
2^{++}	$A\bar{A}$	3968	4209	4080

TABLE III: Thresholds for open charm decays and nearby hidden-charm thresholds.

Channel	Threshold (MeV)	Channel	Threshold (MeV)	Channel	Threshold (MeV)
$D^0\bar{D}^0$	3729.4	$D_s^+D_s^-$	3936.2	$D^0D_s^\pm$	3832.9
D^+D^-	3738.8	$\eta'J/\psi$	4054.7	$D^\pm D_s^\mp$	3837.7
$D^0\bar{D}^{*0}$	3871.3	$D_s^\pm D_s^{*\mp}$	4080.0	$D^{*0}D_s^\pm$	3975.0
$\rho J/\psi$	3872.7	$\phi J/\psi$	4116.4	$D^0D_s^{*\pm}$	3976.7
$D^\pm D^{*\mp}$	3879.5	$D_s^{*+}D_s^{*-}$	4223.8	$K^{*\pm}J/\psi$	3988.6
$\omega J/\psi$	3879.6			$K^{*0}J/\psi$	3993.0
$D^{*0}\bar{D}^{*0}$	4013.6			$D^{*0}D_s^{*\pm}$	4118.8

$$X(1^{+-}) = \frac{1}{\sqrt{2}}[(Qq)_{S=0}(\bar{Q}\bar{q}')_{S=1} - (Qq)_{S=1}(\bar{Q}\bar{q}')_{S=0}]$$

$$X(1^{+-'}) = (Qq)_{S=1}(\bar{Q}\bar{q}')_{S=1}$$

- One state with $J^{PC} = 2^{++}$:

$$X(2^{++}) = (Qq)_{S=1}(\bar{Q}\bar{q}')_{S=1}.$$

The orbitally excited ($1P, 1D \dots$) states are constructed analogously. As we find, a very rich spectrum of tetraquarks emerges. However the number of states in the considered diquark-antidiquark picture is significantly less than in the genuine four-quark approach.

The diquark-antidiquark model of heavy tetraquarks predicts the existence of a flavour $SU(3)$ nonet of states with hidden charm or beauty ($Q = c, b$): four tetraquarks $[(Qq)(\bar{Q}\bar{q})]$, $q = u, d$] with neither open or hidden strangeness, which have electric charges 0 or ± 1 and isospin 0 or 1; four tetraquarks $[(Qs)(\bar{Q}\bar{q})]$ and $(Qq)(\bar{Q}\bar{s})$, $q = u, d$] with open strangeness ($S = \pm 1$), which have electric charges 0 or ± 1 and isospin $\frac{1}{2}$; one tetraquark $(Qs)(\bar{Q}\bar{s})$ with hidden strangeness and zero electric charge. Since we neglect in our model the mass difference of u and d quarks and electromagnetic interactions, the corresponding tetraquarks will be degenerate in mass. A more detailed analysis [12] predicts that the tetraquark mass differences can be of a few MeV so that the isospin invariance is broken for the $(Qq)(\bar{Q}\bar{q})$ mass eigenstates and thus in their strong decays. The (non)observation of such states will be a crucial test of the tetraquark model.

TABLE IV: Masses of charm diquark-antidiquark excited $1P$, $2S$ states (in MeV). S and A denote scalar and axial vector diquarks; \mathcal{S} is the total spin of the diquark and antidiquark. (C is defined only for $q = q'$).

State J^{PC}	Diquark content	\mathcal{S}	Mass		
			$cq\bar{c}\bar{q}$	$cs\bar{c}\bar{s}$	$cq\bar{c}\bar{s}$
$1P$					
1^{--}	$S\bar{S}$	0	4244	4466	4350
$0^{-\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	1	4269	4499	4381
$1^{-\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	1	4284	4514	4396
$2^{-\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	1	4315	4543	4426
1^{--}	$A\bar{A}$	0	4350	4582	4461
0^{-+}	$A\bar{A}$	1	4304	4540	4419
1^{-+}	$A\bar{A}$	1	4345	4578	4458
2^{-+}	$A\bar{A}$	1	4367	4598	4478
1^{--}	$A\bar{A}$	2	4277	4515	4393
2^{--}	$A\bar{A}$	2	4379	4610	4490
3^{--}	$A\bar{A}$	2	4381	4612	4492
$2S$					
0^{++}	$S\bar{S}$	0	4375	4604	4481
$1^{+\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	1	4431	4665	4542
0^{++}	$A\bar{A}$	0	4434	4680	4547
1^{+-}	$A\bar{A}$	1	4461	4703	4572
2^{++}	$A\bar{A}$	2	4515	4748	4625

The calculated masses of the heavy tetraquark ground ($1S$) states and the corresponding open charm thresholds are shown in Tables II, III. Note that most of the tetraquark states were predicted to lie either above or only slightly below corresponding open charm thresholds. In Table IV, V we give our predictions for the orbitally and radially excited tetraquark states with hidden charm. Excitations only of the diquark-antidiquark system are considered. A very rich spectrum of excited tetraquark states is obtained.

In Table VI we compare our results (EFG) for the masses of the ground and excited charm diquark-antidiquark bound states with the predictions of Refs. [12, 13, 14, 15] and with the masses of the recently observed highly-excited charmonium-like states [1, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25]. We assume that the excitations occur only between the bound diquark and antidiquark. Possible excitations of diquarks are not considered. Our calculation of the heavy baryon masses supports such a scheme [11]. In this table we give our predictions only for some of the masses of the orbitally and radially excited states for which possible experimental candidates are observed. The differences in some of the presented theoretical mass values can be attributed to the substantial distinctions in the used approaches. We describe the diquarks dynamically as quark-quark bound systems and calculate their masses and form factors, while in Refs.[12, 13, 14, 15] they are treated only phenomenologically. Then we consider the tetraquark as purely the diquark-antidiquark bound system. In distinction, Maini et al. consider a hyperfine interaction between all

TABLE V: Masses of charm diquark-antidiquark excited $1D$, $2P$ states (in MeV). S and A denote scalar and axial vector diquarks; S is the total spin of the diquark and antidiquark.

State J^{PC}	Diquark content	\mathcal{S}	Mass		
			$cq\bar{c}\bar{q}$	$cs\bar{c}\bar{s}$	$cq\bar{c}\bar{s}$
$1D$					
2^{++}	$S\bar{S}$	0	4506	4728	4611
$1^{+\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	1	4553	4779	4663
$2^{+\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	1	4559	4785	4670
$3^{+\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	1	4570	4794	4680
2^{++}	$A\bar{A}$	0	4617	4847	4727
1^{+-}	$A\bar{A}$	1	4604	4835	4714
2^{+-}	$A\bar{A}$	1	4616	4846	4726
3^{+-}	$A\bar{A}$	1	4624	4852	4733
0^{++}	$A\bar{A}$	2	4582	4814	4692
1^{++}	$A\bar{A}$	2	4593	4825	4703
2^{++}	$A\bar{A}$	2	4610	4841	4720
3^{++}	$A\bar{A}$	2	4627	4855	4736
4^{++}	$A\bar{A}$	2	4628	4856	4738
$2P$					
1^{--}	$S\bar{S}$	0	4666	4884	4767
$0^{-\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	1	4684	4909	4792
$1^{-\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	1	4702	4926	4810
$2^{-\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	1	4738	4960	4845
1^{--}	$A\bar{A}$	0	4765	4991	4872
0^{-+}	$A\bar{A}$	1	4715	4946	4826
1^{-+}	$A\bar{A}$	1	4760	4987	4867
2^{-+}	$A\bar{A}$	1	4786	5011	4892
1^{--}	$A\bar{A}$	2	4687	4920	4799
2^{--}	$A\bar{A}$	2	4797	5022	4903
3^{--}	$A\bar{A}$	2	4804	5030	4910

quarks which, e.g., causes the splitting of 1^{++} and 1^{+-} states arising from the SA diquark-antidiquark compositions. From Table VI we see that our dynamical calculation supports the assumption [12] that $X(3872)$ can be the axial vector 1^{++} tetraquark state composed from the scalar and axial vector diquark and antidiquark in the relative $1S$ state. Recent Belle and BaBar results indicate the existence of a second $X(3875)$ particle a few MeV above $X(3872)$. This state could be naturally identified with the second neutral particle predicted by the tetraquark model [13]. On the other hand, in our model the lightest scalar 0^{++} tetraquark is predicted to be above the open charm threshold $D\bar{D}$ and thus to be broad, while in the model [12] it lies a few MeV below this threshold, and thus is predicted to be narrow. Our 2^{++} tetraquark also lies higher than the one in Ref.[12], thus making the interpretation of this state as $Y(3943)$ less probable, especially if one averages the original Belle result with the recent BaBar value which is somewhat lower.

TABLE VI: Comparison of theoretical predictions for the masses of the ground and excited charm diquark-antidiquark states (in MeV) and possible experimental candidates.

State J^{PC}	Diquark content	Theory			Experiment	
		EFG	[12, 13, 14]	[15] ($cs\bar{c}\bar{s}$)	state	mass
$1S$						
0^{++}	$S\bar{S}$	3812	3723			
1^{++}	$(S\bar{A} + \bar{S}A)/\sqrt{2}$	3871	3872 [†]		$\left\{ \begin{array}{l} X(3872) \\ X(3876) \end{array} \right\}$	$\left\{ \begin{array}{l} 3871.4 \pm 0.6 [1] \\ 3875.2 \pm 0.7_{-1.8}^{+0.9} [1] \end{array} \right\}$
1^{+-}	$(S\bar{A} - \bar{S}A)/\sqrt{2}$	3871	3754			
0^{++}	$A\bar{A}$	3852	3832			
1^{+-}	$A\bar{A}$	3890	3882			
2^{++}	$A\bar{A}$	3968	3952		$Y(3943)$	$\left\{ \begin{array}{l} 3943 \pm 11 \pm 13 [16] \\ 3914.3_{-3.8}^{+4.1} [17] \end{array} \right\}$
$1P$						
1^{--}	$S\bar{S}$	4244		4330 \pm 70	$Y(4260)$	$\left\{ \begin{array}{l} 4259 \pm 8_{-6}^{+2} [18] \\ 4247 \pm 12_{-32}^{+17} [19] \end{array} \right\}$
1^{-}	$S\bar{S}$	4244	$\left. \begin{array}{l} \end{array} \right\}$		$Z(4248)$	4248 $_{-29-35}^{+44+180}$ [20]
0^{-}	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	4267				
1^{--}	$(S\bar{A} - \bar{S}A)/\sqrt{2}$	4284	$\left. \begin{array}{l} \end{array} \right\}$		$Y(4260)$	4284 $_{-16}^{+17} \pm 4$ [21]
1^{--}	$A\bar{A}$	4277				
1^{--}	$A\bar{A}$	4350			$Y(4360)$	$\left\{ \begin{array}{l} 4361 \pm 9 \pm 9 [22] \\ 4324 \pm 24 [23] \end{array} \right\}$
$2S$						
1^{+}	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	4431	$\left. \begin{array}{l} \end{array} \right\}$		$Z(4430)$	4433 \pm 4 \pm 2 [24]
0^{+}	$A\bar{A}$	4434				
1^{+}	$A\bar{A}$	4461		~ 4470		
$2P$						
1^{--}	$S\bar{S}$	4666			$\left\{ \begin{array}{l} Y(4660) \\ X(4630) \end{array} \right\}$	$\left\{ \begin{array}{l} 4664 \pm 11 \pm 5 [22] \\ 4634_{-7-8}^{+8+5} [25] \end{array} \right\}$

[†] input

The recent discovery in the initial state radiation at B -factories of the $Y(4260)$, $Y(4360)$ and $Y(4660)$ indicates an overpopulation of the expected charmonium 1^{--} states [1, 18, 19, 21, 22, 23]. Maini et al. [15] argue that $Y(4260)$ is the 1^{--} $1P$ state of the charm-strange diquark-antidiquark tetraquark. We find that $Y(4260)$ cannot be interpreted in this way, since the mass of such ($[cs]_{S=0}[\bar{c}\bar{s}]_{S=0}$) tetraquark is found to be ~ 200 MeV higher. A more natural tetraquark interpretation could be the 1^{--} $1P$ state ($[cq]_{S=0}[\bar{c}\bar{q}]_{S=0}$) ($S\bar{S}$) which mass is predicted in our model to be close to the mass of $Y(4260)$ (see Table VI). Then the $Y(4260)$ would decay dominantly into $D\bar{D}$ pairs. The other possible interpretations of $Y(4260)$ are the 1^{--} $1P$ states of $(S\bar{A} - \bar{S}A)/\sqrt{2}$ and $A\bar{A}$ tetraquarks which predicted masses have close values. These additional tetraquark states could be responsible for the

mass difference of $Y(4260)$ observed in different decay channels. As we see from Table VI, the recently discovered resonances $Y(4360)$ and $Y(4660)$ in the $e^+e^- \rightarrow \pi^+\pi^-\psi'$ cross section can be interpreted as the excited $1^{--} 1P$ ($A\bar{A}$) and $2P$ ($S\bar{S}$) tetraquark states, respectively. The peak $X(4630)$ very recently observed by Belle in $e^+e^- \rightarrow \Lambda_c^+\Lambda_c^-$ [25] is consistent with a 1^{--} resonance $Y(4660)$ and therefore has the same interpretation in our model.

Recently the Belle Collaboration reported the observation of a relatively narrow enhancement in the $\pi^+\psi'$ invariant mass distribution in the $B \rightarrow K\pi^+\psi'$ decay [1, 24]. This new resonance, $Z^+(4430)$, is unique among other exotic meson candidates, since it is the first state which has a non-zero electric charge. Different theoretical interpretations were suggested [1]. Maiani et al. [14] give qualitative arguments that the $Z^+(4430)$ could be the first radial excitation ($2S$) of a diquark-antidiquark $X_{ud}^+(1^{+-}; 1S)$ state ($A\bar{A}$) with mass 3882 MeV. Our calculations indicate that the $Z^+(4430)$ can indeed be the $1^+ 2S$ $[cu][\bar{c}\bar{d}]$ tetraquark state. It could be the first radial excitation of the ground state $(S\bar{A} - \bar{S}A)/\sqrt{2}$, which has the same mass as $X(3872)$. The other possible interpretation is the $0^+ 2S$ $[cu][\bar{c}\bar{d}]$ tetraquark state ($A\bar{A}$) which has a very close mass. Measurement of the $Z^+(4430)$ spin will discriminate between these possibilities.

Encouraged by this discovery, the Belle Collaboration performed a study of $\bar{B}^0 \rightarrow K^-\pi^+\chi_{c1}$ and observed a double peaked structure in the $\pi^+\chi_{c1}$ invariant mass distribution [20]. These two charged hidden charm peaks, $Z(4051)$ and $Z(4248)$, are explicitly exotic. We find no tetraquark candidates for the former, $Z(4051)$, structure. On the other hand, we see from Table VI that $Z(4248)$ can be interpreted in our model as the charged partner of the $1^- 1P$ state $S\bar{S}$ or as the $0^- 1P$ state of the $(S\bar{A} \pm \bar{S}A)/\sqrt{2}$ tetraquark.

In summary, we calculated the masses of excited heavy tetraquarks with hidden charm in the diquark-antidiquark picture. In contrast to previous phenomenological treatments, we used the dynamical approach based on the relativistic quark model. Both diquark and tetraquark masses were obtained by numerical solution of the quasipotential wave equation with the corresponding relativistic potentials. The diquark structure was taken into account in terms of diquark wave functions. It is important to emphasize that, in our analysis, we did not introduce any free adjustable parameters but used their values fixed from our previous considerations of heavy and light hadron properties. It was found that the $X(3872)$, $Z(4248)$, $Y(4260)$, $Y(4360)$, $Z(4430)$ and $Y(4660)$ exotic meson candidates can be tetraquark states with hidden charm.

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